AD-A131 917 STABILITY OF A PAIR OF STATIONARY VORTICES IN THE LEEWARD SIDE OF A CYLINDER IN A POTENTIAL FLOWFIELD(U) DAYTON UNIV OH RESEARCH INST S N SINGH MAY 83 1/1

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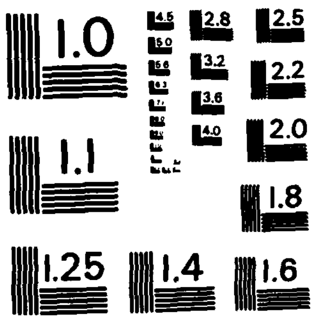
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STABILITY OF A PAIR OF STATIONARY VORTICES IN THE LEEWARD SIDE OF A CYLINDER IN A POTENTIAL FLOWFIELD

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May 1983
Final Technical Report - May 1981-September 1981

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REPORT DOCUMENTATION PAGE

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1. REPORT NUMBER AFWAL-TR-83-3067	2. GOVT ACCESSION NO. AD-A131 917	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) STABILITY OF A PAIR OF STATIONARY VORTICES IN THE LEEWARD SIDE OF A CYLINDER IN A POTENTIAL FLOWFIELD		5. TYPE OF REPORT & PERIOD COVERED Final Technical Report May 1981-Sep 1981 6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) Shiva N. Singh		8. CONTRACT OR GRANT NUMBER(s) F33615-79-C-3030
9. PERFORMING ORGANIZATION NAME AND ADDRESS University of Dayton Research Institute Dayton, Ohio		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS 61102F 2307N603
11. CONTROLLING OFFICE NAME AND ADDRESS Flight Dynamics Laboratory (AFWAL/FIMM) Air Force Wright Aeronautical Laboratories Wright-Patterson AFB, OH 45433		12. REPORT DATE 13. NUMBER OF PAGES 18
14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office)		15. SECURITY CLASS. (of this report) UNCLASSIFIED 15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Vortex Flows
Stability
Cylinder Flow Field

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

In this report the steady state flow of an incompressible inviscid fluid past a circular cylinder is considered when a pair of asymmetrical stationary vortices are situated in the rear of the cylinder. Complex potential for this flow is obtained with the help of Milne-Thompson's circle theorem. The condition that the vortex centers have zero induced velocity determines the positions of the stationary vortices. This condition is determined for the various combination of the free stream velocity, the radius of the cylinder,

and the strengths of the vortices by the secant search routine on the computer. Asymmetrical disturbances are introduced and a stability analysis of the system is performed. And finally for the stable systems, the lift force acting on the cylinder is calculated and compared with the experimental results.

ACKNOWLEDGEMENT

The author would like to thank the Air Force Systems Command for providing him with the opportunity to spend a very worthwhile and interesting seven weeks at Wright-Patterson Air Force Base, Ohio. He would like to acknowledge the Flight Dynamics Laboratory, in particular the Computational Aerodynamics Group, for its hospitality and excellent working conditions and the staff at the University of Dayton Research Institute for their generous help.

He would like to thank Dr. Wilbur Hankey for suggesting this area of research and for his collaboration and guidance. The generous computational help given by Mr. Steve Scherr in programming the problem on the computer and several helpful discussions with Dr. G. R. Verma are much appreciated.

TABLE OF CONTENTS

SECTION 1

INTRODUCTION

During the last several years, aerodynamics of missiles at large incidence has received considerable attention because of atmospheric flight requirements such as high maneuverability and high launch angles of attack. Nonlinear forces and moments which come into existence at high angles of incidence have caused a number of flight stability and controllability problems on both missiles and aircraft. Extensive schlieren studies and yawmeter traverses of the wake behind slender cone-cylinders at large angles of incidence have shown that the flow pattern is generally steady. Under certain flow conditions, however, the wake exhibits an instability which is not understood. Close to the body nose no wake exists whereas further downstream two symmetrically disposed vortices form on the lee side. These vortices are fed by vortex sheets containing boundary layer fluid which has separated from the body. Further along the body first one and then the other of these vortices detaches and moves downstream at an angle to the free stream. At moderate angles of incidence, a pair of stationary symmetric vortices appear on the leeward side and the symmetrical wake configuration predominates. At large angles of incidence this shrinks towards the nose and an asymmetric flow pattern becomes very extensive. At even higher angles of attack the shedding vortices become completely unsteady. The steady asymmetric case is of considerable interest to the aircraft industry since large side forces result at zero yaw angles capable of producing aircraft departure into a spin situation.

To study the experimental data, Jorgensen and Perkins[1] constructed a simple theoretical model in which the induced flow field in any crossflow plane along the cylindrical afterbody is represented by the incompressible steady potential flow around a cylinder in the presence of two symmetrical vortices of equal strength. Based on the model, vortex paths, which agree well with the experimental paths have been computed. A more extensive study of the symmetric vortex wake was presented by Mello[2], who measured total pressure and cross-flow velocity on a cone-cylinder body at Mach Number 2. In subsonic flow, body vortex wake has been studied by Tinling and Allen[3], Fiechter[4], Grosche[5], Fidler, Nielsen and Schwind[6], Yanta and Wardlaw[7], and Owen and Johnson[8]. Thompson and Morrison[9] analyzed the schlieren photographs of the wake by means of the impulse flow analogy and also by considering the vortices to be a part of a yawed infinite vortex sheet. The impulse flow analogy is shown to be of use in determing the cross-flow Strouhal number but estimates of vortex strength are too high. The Karman vortex street theory combined with sweepback principle leads to reliable estimates of vortex strength up to the cross flow Mach number equal to unity. The pressure and force distribution data have been presented by Lamont and Hunt[10] on a sharp-nosed circular cylinder at large angles of inclination to a uniform subsonic stream.

The symmetric body vortex wake of a circular cylinder in supersonic flow has been investigated by Oberkampf and Bartel[11]. They measured the flow field in the cross flow plane at various axial body stations for angles of attack from 10 to 25 degrees. Recently Hankey, Graham, and Shang[12] presented a solution of the three dimensional flow field surrounding an ogive-cylinder at various angles of attack through integration of the Navier-Stokes equations on a CRAY-1 computer. The MacCormack's algorithm vectorized by Shang[13] was used. The computed results compare favorably with experimental data in that all primary features are captured. The flow on the leeward side of a slender body of revolution has been experimentally investigated by Calarese[14] at large incidence for Mach numbers equal to 0·6 and 0·7 and unit Reynolds number of $2{\cdot}4 \times 10^6$. Hot film anemometers have been used to obtain the data. The asymmetry and the steadiness of the vortex shedding pattern is ascertained, Shivananda and Oberkampf[15] used the impulse flow analogy in conjunction with the discrete vortex method to calculate theoretically the symmetric and asymmetric body vortex wake. Agreement between the calculated values of body forces and moments and those experimentally measured for both subsonic and supersonic free stream flow was satisfactory.

The present study considers the potential flow past a circular cylinder of radius, a, in the presence of two point vortices of strengths K_1 and K_2. These vortices are located in the complex $Z(=x + iy)$ plane at the two arbitrary points Z_1 and Z_2 respectively. Two parameters govern the problem $2\pi Ua/K_1$ and K_2/K_1 where U is the magnitude of the free stream velocity. For several combination of these two parameters, values of Z_1 and Z_2 are found such that the induced velocity of the vortex centers is zero. Stability criteria are then applied to ascertain whether the system is stable. And for the stable system the lift and drag forces acting on the cylinder are calculated. Finally, these results are compared with those available experimental studies under similar conditions.

SECTION II

MATHEMATICAL ANALYSIS

Consider an inviscid incompressible fluid flowing from left to right with free stream velocity U past a fixed cylinder of radius, a, the origin being at the center of the cylinder. There are two point vortices of strengths $-K_1$ and K_2 located at points $A(Z_1)$ and $B(Z_2)$ outside the cylinders in the leeward side as shown in Figure 1.* The complex potential for the flow can be obtained with the help of circle theorem (see Milne-Thompson[16], pages 336-370).

$$W(Z) = -U(Z + \frac{a^2}{Z}) - \frac{iK_1}{2\pi} \ln (Z - Z_1) + \frac{iK_1}{2\pi} \ln (Z - \frac{a^2}{\overline{Z}_1}) .$$

$$- \frac{i(K_1 - K_2)}{2\pi} \ln Z + \frac{iK_2}{2\pi} \ln (Z - Z_2) - \frac{iK_2}{2\pi} \ln (Z - \frac{a^2}{\overline{Z}_2}) \quad (1)$$

where $\overline{Z}_1$ and $\overline{Z}_2$ are the complex conjugate of Z_1 and Z_2 respectively. Hence the motion of the vortex A is obtained from the function

$$W_{Z_1} = -U(Z + \frac{a^2}{Z^2}) + \frac{iK_1}{2\pi} \ln (Z - \frac{a^2}{\overline{Z}_1}) - \frac{i(K_1 - K_2)}{2\pi} \ln Z$$

$$+ \frac{iK_2}{2\pi} \ln (Z - Z_2) - \frac{iK_2}{2\pi} \ln (Z - \frac{a^2}{\overline{Z}_2}) \quad (2)$$

The vortex A will be at rest if $(dW_{Z1}/dZ) = 0$ when $Z = Z_1$. Differentiating (2) with respect Z and putting $Z = Z_1$ we obtain

$$\frac{dW_{Z1}}{dZ} = - U (1 - \frac{a^2}{Z_1^2}) + \frac{iK_1}{2\pi} \frac{\overline{Z}_1}{(Z_1\overline{Z}_1 - a^2)} - \frac{i(K_1 - K_2)}{2\pi Z_1}$$

$$+ \frac{iK_2}{2\pi} \frac{1}{(Z_1 - Z_2)} - \frac{iK_2}{2\pi} \frac{\overline{Z}_2}{(Z_1\overline{Z}_2 - a^2)} = 0 \quad (3)$$

Similarly the condition that the vortex B is stationary can be obtained as

$$\frac{dW_{Z_2}}{dZ} = - U (1 - \frac{a^2}{Z_2^2}) - \frac{iK_1}{2\pi (Z_2 - Z_1)} + \frac{iK_1\overline{Z}_1}{2\pi (Z_2\overline{Z}_1 - a^2)}$$

$$- \frac{i(K_1 - K_2)}{2\pi Z_2} - \frac{iK_2\overline{Z}_2}{2\pi(Z_2\overline{Z}_2 - a^2)} = 0 \quad (4)$$

*Figures are located at end of report.

The conditions (3) and (4) for the vortices A and B to have zero velocity relative to the cylinder can be put in the non-dimensional form if Z_1, $\bar{Z}_1$, Z_2, and $\bar{Z}_2$ are divided by a

$$-\alpha\left(1-\frac{1}{\zeta_1^2}\right)+\frac{i\bar{\zeta}_1}{(\zeta_1\bar{\zeta}_1-1)}-\frac{i\,(1-\beta)}{\zeta_1}+\frac{i\beta}{(\zeta_1-\zeta_2)}-\frac{i\bar{\zeta}_2\beta}{(\zeta_1\bar{\zeta}_2-1)}=0 \qquad (5)$$

and

$$-\alpha\left(1-\frac{1}{\zeta_2^2}\right)-\frac{i}{(\zeta_2-\zeta_1)}+\frac{i\bar{\zeta}_1}{(\zeta_2\bar{\zeta}_1-1)}-\frac{i(1-\beta)}{\zeta_2}-\frac{i\beta\bar{\zeta}_2}{(\zeta_2\bar{\zeta}_2-1)}=0 \qquad (6)$$

where

$$\zeta_1 = Z_1/a,\ \bar{\zeta}_1 = \bar{Z}_1/a,\ \zeta_2 = Z_2/a,\ \bar{\zeta}_2 = Z_2/a \text{ and } \alpha = 2\pi aU/K_1,$$
$$\beta = K_2/K_1$$

For given values of α and β, equations (5) and (6) can be separated into real and imaginary parts. Thus, we get four equations for four unknowns ξ_1,η_1 and ξ_2,η_2 where $\zeta_1 = \xi_1 + i\eta_1$, etc, and these determine the positions of the vortices such that the vortices are stationary.

To examine the stability of the above-mentioned system, it is observed that at time (t) the two vortices are at points Z_1 and Z_2 respectively. If each vortex is displaced slightly, the positions of the vortices will be given by $Z_1 + Z_m$ and $Z_2 + Z_n$ respectively where $|Z_m|$ and $|Z_n|$ are both small initially. The system will be stable if these quantities decay exponentially.

The complex velocity of the vortex A in the displaced position is given by

$$\frac{dW_{Z_1}}{dZ}+\frac{d\bar{Z}_m}{dt}=-U\left[1-\frac{a^2}{(Z_1+Z_m)^2}\right]+\frac{iK_1(\bar{Z}_1+\bar{Z}_m)/2\pi}{(Z_1+Z_m)\,(Z_1+Z_m)-a^2}-\frac{i(K_1-K_2)/2\pi}{Z_1+Z_m}$$

$$+\frac{iK_2/2\pi}{(Z_1+Z_m-Z_2-Z_n)}-\frac{iK_2(\bar{Z}_2+\bar{Z}_n)/2\pi}{(Z_1+Z_m)\,(\bar{Z}_2+\bar{Z}_n)-a^2} \qquad (7)$$

Since both $|Z_m|$ and $|Z_n|$ are small initially, equation (7) becomes making use of the condition (3),

$$\frac{d\bar{Z}_m}{dt}=A_1Z_m+B_1\bar{Z}_m+C_1Z_n+D_1\bar{Z}_n \qquad (8)$$

where

$$2\pi A_1=\frac{-4\pi Ua^2}{Z_1^3}-\frac{iK_2}{(Z_1-Z_2)^2}-\frac{iK_1\bar{Z}_1^2}{(Z_1\bar{Z}_1-a^2)^2}+\frac{iK_2\bar{Z}_2^2}{(Z_1\bar{Z}_2-a^2)^2}+\frac{i(K_1-K_2)}{Z_1^2}$$

$$2\pi B_1 = \frac{-iK_1a^2}{(Z_1\overline{Z}_1-a^2)^2} \quad , \quad 2\pi C_1 = \frac{+iK_2}{(Z_1-Z_2)^2} \quad , \quad 2\pi D_1 = \frac{iK_2a^2}{(Z_1\overline{Z}_2-a^2)^2}$$

Similarly the complex velocity of the displaced vortex B can be obtained as:

$$\frac{d\overline{Z}_n}{dt} = A_2Z_m + B_2\overline{Z}_m + C_2Z_n + D_2\overline{Z}_n \tag{9}$$

where

$$2\pi A_2 = \frac{-iK_1}{(Z_1-Z_2)^2} \quad , \quad 2\pi B_2 = \frac{-iK_1a^2}{(Z_2\overline{Z}_1-a^2)^2} \quad , \quad 2\pi D_2 = \frac{iK_2a^2}{(Z_2\overline{Z}_2-a^2)^2}$$

$$2\pi C_2 = \frac{4\pi Ua^2}{Z_2{}^3} + \frac{iK_1}{(Z_2-Z_1)^2} - \frac{iK_1\overline{Z}_1{}^2}{(Z_2\overline{Z}_1-a^2)^2} + \frac{i(K_1-K_2)}{Z_2{}^2} + \frac{iK_2\overline{Z}_2{}^2}{(Z_2\overline{Z}_2-a^2)^2}$$

Taking the conjugate of equations (8) and (9), we obtain:

$$\frac{dZ_m}{dt} = \overline{A}_1\overline{Z}_m + \overline{B}_1Z_m + \overline{C}_1\overline{Z}_n + \overline{D}_1Z_n \tag{10}$$

and

$$\frac{dZ_n}{dt} = \overline{A}_2\overline{Z}_m + \overline{B}_2Z_m + \overline{C}_2\overline{Z}_n + \overline{D}_2Z_n \tag{11}$$

where $\overline{A}_1$, $\overline{B}_1, \ldots, \overline{D}_2$, etc. are the complex conjugate of A_1, $B_1, \ldots,$ and D_2.

The stability problem is well set. We can assume

$$Z_m = S_1 \exp(\lambda t) \quad , \quad \overline{Z}_m = \overline{S}_1 \exp(\lambda t)$$

$$Z_n = S_3 \exp(\lambda t) \quad , \quad \overline{Z}_n = \overline{S}_3 \exp(\lambda t) \tag{12}$$

and substitute (12) into (8) through (11). If anyone of the λ roots possesses a positive real part, the system will be unstable. If λ has negative real parts, then the system is stable.

In this report only antisymmetric disturbances are considered, i.e., Z_n and $\overline{Z}_n$ are assumed to be zero. In this case, we get

$$\frac{d\overline{Z}_m}{dt} = A_1Z_m + B_1\overline{Z}_m \quad , \quad \frac{dZ_m}{dt} = \overline{A}_1\overline{Z}_m + \overline{B}_1Z_m \tag{13}$$

or

$$\left[\frac{d^2}{dt^2} - (B_1 + \overline{B}_1)\frac{d}{dt} + (B_1\overline{B}_1 - A_1\overline{A}_1)\right](Z_m, \overline{Z}_m) = 0 \tag{14}$$

Since B_1 is purely imaginary, $B_1 + \overline{B}_1$ is zero. Solution of (14) is given by the exponential form similar to (12). Hence

$$\lambda = \pm \{A_1\overline{A}_1 - B_1\overline{B}_1\}^{1/2} \tag{15}$$

Conditions for the stability become

$$(A_1\overline{A}_1 - B_1\overline{B}_1) < 0 \tag{16}$$

To calculate the lift and drag forces acting on the cylinder, we can use the Blasius theorem (see Milne-Thompson[15], page 173). If X and Y are the components of the forces acting on the body, one gets

$$X - iY = \frac{1}{2}\rho i \oint \left(\frac{dW}{dZ}\right)^2 dZ \tag{17}$$

The path of integration for the integral in (17) is the body surface. Equation (1) gives

$$2\pi\frac{dW}{dZ} = -2\pi U\left(1 - \frac{a^2}{Z^2}\right) - \frac{iK_1}{Z - Z_1} + \frac{iK_1\overline{Z}_1}{(Z\overline{Z}_1 - a^2)} - \frac{i(K_1 - K_2)}{Z} + \frac{iK_2}{Z - Z_2} - \frac{iK_2\overline{Z}_2}{(Z\overline{Z}_2 - a^2)} \tag{18}$$

When (18) is substituted into (17), we find that there are three poles inside the circle at the origin, $a^2/\overline{Z}_1$ and $a^2/\overline{Z}_2$. Hence, (17) gives

$$\frac{X - iY}{\frac{1}{2}\rho U^2 2\pi a} = \left[\frac{2i}{\alpha}\left(\frac{\beta}{\zeta_2^2} - \frac{1}{\zeta_1^2}\right)\right] - \frac{2}{\alpha^2}\left[\frac{\overline{\zeta}_1}{1 - \zeta_1\overline{\zeta}_1} + \frac{1 - \beta}{\zeta_1} - \frac{\beta\overline{\zeta}_2}{1 - \zeta_1\overline{\zeta}_2}\right]$$

$$+ \frac{2\beta}{\alpha^2}\left[\frac{\overline{\zeta}_1}{1 - \overline{\zeta}_1\zeta_2} + \frac{1 - \beta}{\zeta_2} - \frac{\beta\overline{\zeta}_2}{1 - \zeta_2\overline{\zeta}_2}\right] = C_x - iC_y = \frac{C_D - iC_L}{\pi} \tag{19}$$

For given values of α and β, the roots of (5) and (6) find the location of the zero-velocity vortices. The criterion (16) finds out whether the above-mentioned determined system is stable. Finally the forces acting on the cylinder are calculated by the equation (19).

SECTION III

DISCUSSION OF RESULTS

Foppl[17] investigated mathematically the motion of a vortex-pair symmetrically placed behind a circular cylinder in a uniform stream. His results could be derived if in the above analysis, we substitute that

$$K_1 = K_2 \ , \quad Z_2 = \overline{Z}_1 \ , \quad \overline{Z}_2 = Z_1 \ , \beta = 1 \ , \quad \overline{\zeta}_2 = \zeta_1 \ , \quad \zeta_2 = \overline{\zeta}_1 \tag{20}$$

With this simplification, equations (3) and (4) reduce to (see also Milne-Thompson[16], page 370)

$$(Z_1\overline{Z}_1 - a^2)^2 + Z_1\overline{Z}_1(Z_1 - \overline{Z}_1)^2 = 0 \tag{21}$$

Putting $Z_1 = \gamma_1 e^{i\theta_1}$, where $0 < \theta < \pi/2$, this gives

$$\left(\frac{\gamma_1}{a} - \frac{a}{\gamma_1}\right) = \frac{2\gamma_1}{a} \sin\theta_1 \ , \quad \frac{K_1}{2\pi Ua} = \frac{2\gamma_1 \sin\theta}{a} \left(1 - \frac{a^4}{\gamma_1^4}\right) \tag{22}$$

Equations (22) are exactly the same as (7) and (8) in Foppl's paper. To verify Foppl's results regarding the stability of the system, equations (8) to (11) are invoked. We note that

$$A_1 = \overline{C}_2 \ , \quad B_1 = \overline{D}_2 \ , \quad C_1 = \overline{A}_2 \ , \quad D_1 = \overline{B}_2 \ . \tag{23}$$

Four equations (8) to (11) reduce to

$$\frac{d}{dt}(Z_m + \overline{Z}_n) = (\overline{A}_1 + \overline{D}_1)(\overline{Z}_m + Z_n) + (\overline{B}_1 + \overline{C}_1)(Z_m + \overline{Z}_n) \tag{24}$$

and

$$\frac{d}{dt}(\overline{Z}_m + Z_n) = (A_1 + D_1)(Z_m + \overline{Z}_n) + (B_1 + C_1)(\overline{Z}_m + Z_n) \tag{25}$$

The corresponding stability conditior similar to the (16) is obtained as

$$(A_1 + D_1)\ (\overline{A}_1 + \overline{D}_1) - (B_1 + C_1)\ (\overline{B}_1 + \overline{C}_1) < 0 \tag{26}$$

It is found that for all the combinations of $K_1/2\pi Ua$ and θ, given by (22) the condition (26) is satisfied. In otherwords the system is found to be stable for symmetrical disturbances. On the other hand when $Z_n = \overline{Z}_n = o$ and the parameters are given by (22), we find that for $(\gamma_1/a) \leq 1\cdot209$, the stability condition given by (16) is satisfied. But for $(\gamma_1/a) > 1\cdot209$, the system becomes unstable. Hence the symmetrical portion of the pair of vortices is found to be unstable for antisymmetrical disturbances when $(\gamma_1/a) > 1\cdot209$.

Further for the symmetrically situated vortex pair of equal and opposite strengths, the force components acting on the cylinder have been calculated by Muller[17] and Bickley[18]. For this case the expression (19) becomes

$$\frac{X - iY}{\frac{1}{2}\rho U^2 2\pi a} = \frac{2i}{\alpha}\left(\frac{1}{\overline{\zeta}_1^2} - \frac{1}{\zeta_1^2}\right) - \frac{2}{\alpha^2}\left[\frac{\overline{\zeta}_1}{1 - \zeta_1\overline{\zeta}_1} - \frac{\zeta}{1 - \zeta_1^2}\right]$$

$$= + \frac{2}{\alpha^2}\left[\frac{\overline{\zeta}_1}{1-\overline{\zeta}_1^2} - \frac{\zeta_1}{1 - \zeta_1\overline{\zeta}^1}\right] = - \frac{4a^2}{\alpha\gamma_1{}^2}\sin 2\theta\ , + \frac{4a\gamma_1\cos\theta}{\alpha^2(\gamma_1{}^2 - a^2)}$$

$$- \frac{4a\gamma_1\cos\theta\,(\gamma_1^2 - a^2)}{a^4 - 2a^2\gamma_1^2\cos 2\theta + \gamma_1{}^4} = C_X - C_Y = \frac{C_D - iC_L}{\pi} \tag{27}$$

This is the same relation as Bickley's equation (3·21). Thus, it is verified that the analysis presented in this report reduces in the special case to Foppl's results already available in the literature. The locus of the two vortices as given by equation (21) is also shown in Figure 1.

With the help of secant search sub-routine roots of equations (5) and (6) for various values of α and β are investigated. The admissible roots have to be such that

$$(\xi_1^2 + \eta_1^2) > 1, \quad (\xi_2^2 + \eta_2^2) > 1, \text{ and } \eta_2 \leq 0.$$

The condition for the stability (16) has to be satisfied. On the basis of systematic search, values have been obtained and these are given in Table 1. The lift for each case has also been calculated. In Figure 2, the lift coefficients $C_L(=\pi C_y)$ are plotted graphically versus $\beta/\alpha(= K_2/2\pi aU)$ for various values of $1/\alpha(= K_1/2\pi aU)$. It is clear from Fig. 2 that the lift coefficients of the order of 2 or 3 due the stable locations of the vortices are comparable with those obtained by Shivananda and Oberkampf[15]. Although the results given in Table 1 and shown in Fig. 2 are restricted to the values of $1/\alpha$ equal to or less than 0.1, it is found that for values of $\alpha > 10$ (or $1/\alpha < 0.1$), higher values of the lift coefficients C_L can be obtained on the basis of the theoretical study presented in this report. The region of instability is also shown in Fig. 2. The positions of the asymmetrical vortices are shown in Fig. 1 and the regions of the unstable locations are pointed out. Thus the theoretical study based on the ideal inviscid motion proves that the circular cylinder held fixed in a free stream with the asymmetrically situated vortices experiences a lift of the magnitudes comparable with those experimentally measured.

TABLE 1

Location of stationary vortices for various values of α and β

α	β/α	β	ξ_1	η_1	ξ_2	η_2	C_L	Stable
0.42	2.98	1.25	1.44	1.54	2.08	-1.46	-0.069	no
0.5	2.50	1.25	1.67	1.02	2.12	-1.56	-0.832	yes
0.6	2.08	1.25	1.70	0.79	2.02	-1.41	-0.748	yes
0.7	1.79	1.25	1.68	0.67	1.91	-1.23	-0.559	no
0.6	2.50	1.5	1.22	1.27	1.93	-1.25	0.226	yes
0.8	1.88	1.5	1.48	0.69	1.89	-1.31	-0.528	yes
1.0	1.50	1.5	1.49	0.52	1.74	-1.12	-0.286	yes
1.2	1.25	1.5	1.47	0.44	1.64	-0.97	-0.056	yes
1.6	0.94	1.5	1.43	0.35	1.50	-0.76	0.252	no
0.9	2.22	2.0	1.02	1.12	1.82	-1.08	1.005	yes
1.2	1.67	2.0	1.29	0.5	1.79	-1.20	-0.047	yes
2.0	1.00	2.0	1.33	0.30	1.48	-0.89	0.396	yes
2.4	0.833	2.0	1.33	0.25	1.40	-0.78	0.537	yes
3.0	0.667	2.0	1.32	0.20	1.34	-0.66	0.645	no
1.1	2.27	2.5	0.78	1.25	1.69	-0.92	2.849	no
1.8	1.40	2.5	1.24	0.43	1.66	-1.12	0.327	yes
2.6	0.96	2.5	1.27	0.25	1.44	-0.90	0.600	yes
3.4	0.74	2.5	1.27	0.18	1.33	-0.75	0.757	yes
4.3	0.58	2.5	1.28	0.14	1.27	-0.64	0.823	no
1.3	2.31	3.0	0.64	1.30	1.59	-0.86	4.445	no
2.6	1.15	3.0	1.21	0.31	1.53	-1.02	0.625	yes
3.8	0.79	3.0	1.23	0.18	1.34	-0.82	0.842	yes
4.6	0.65	3.0	1.24	0.13	1.28	-0.72	0.905	yes
5.6	0.54	3.0	1.25	0.10	1.24	-0.64	0.908	no

α	β/α	β	ξ_1	η_1	ξ_2	η_2	C_L	Stable
1.8	2.22	4.0	0.63	1.16	1.60	-0.87	3.757	no
3.2	1.25	4.0	1.14	0.35	1.58	-1.07	0.848	yes
4.8	0.83	4.0	1.17	0.17	1.34	-0.87	0.986	yes
6.4	0.62	4.0	1.19	0.09	1.24	-0.74	1.051	yes
8.4	0.48	4.0	1.21	0.05	1.19	-0.71	1.037	no
2.3	2.17	5.0	0.63	1.08	1.62	-0.93	2.786	no
3.6	1.39	5.0	1.07	0.42	1.66	-1.09	1.068	yes
5.6	0.89	5.0	1.13	0.18	1.37	-0.92	1.093	yes
8.0	0.62	5.0	1.16	0.08	1.23	-0.76	1.144	yes
11.0	0.45	5.0	1.19	0.02	1.17	-0.63	1.13	no
2.7	2.22	6.0	0.42	1.31	1.40	-0.84	8.796	no
3.0	2.00	6.0	0.73	0.91	1.70	-0.92	3.079	yes
5.0	1.20	6.0	1.08	0.32	1.56	-1.04	1.159	yes
9.0	0.67	6.0	1.13	0.09	1.24	-0.79	1.213	yes
14.0	0.43	6.0	1.17	0.00	1.15	-0.63	1.123	no
3.7	2.16	8.0	0.58	1.01	1.60	-0.88	4.750	no
5.5	1.45	8.0	0.99	0.47	1.70	-1.06	1.517	yes
8.0	1.0	8.0	1.07	0.22	1.43	-0.97	1.307	yes
12.0	0.67	8.0	1.10	0.08	1.23	-0.80	1.309	yes
20.0	0.40	8.0	1.15	0.02	1.12	-0.62	1.178	no
4.6	2.12	10.0	0.56	0.99	1.59	-0.89	1.589	no
6.0	1.67	10.0	0.88	0.63	1.76	-1.01	2.080	yes
12.0	0.83	10.0	1.06	0.15	1.33	-0.90	1.388	yes
18.0	0.56	10.0	1.09	0.03	1.17	-0.74	1.332	yes
26.0	0.38	10.0	1.14	0.04	1.11	-0.62	1.187	no

SECTION IV

Conclusion

With the aid of complex - potential flow analysis, the location of a pair of asymmetric zero-velocity vortices for flow past a circular cylinder is investigated in terms of two-dimensionless parameters α ($= 2\pi Ua/K_1$) and β ($= K_2/K_1$). The condition for the stability of the system is established by the small perturbation theory. The Blasius theorem determines the forces acting on the system. It is pointed out that if the values of α and β are known for any actual flight conditions, the mathematical analysis presented in this paper will be of great help in analyzing the data.

SECTION V

RECOMMENDATION

The action of viscosity and the non-linear interaction on the stability of two infinite vortex sheets have been examined by several authors[21, 22, 23]. Similary the effects of viscosity and non-linearity can be investigated on the stability of a pair of vorticity in the leeward side of a cylinder. In the present investigation, the vortices were taken to be point-vortices. In the future work, an attempt can be made to study the effect on the configuration of vortices in the wake behind a cylinder of an allowance for the thickness of the vortices in the potential flow field. Schlayer[24] and Rosenhead[25] presented such studies for the stability of the Karman double row of recti-linear vortices.

REFERENCES

[1]Jorgensen, L. H. and E. W. Perkins, "Investigation of Some Wake Vortex Characteristics of an Inclined Ogive-Cylinder Body at Mach No. 2," NACA Report 1371, May 1955.

[2]Mello, J. F., "Investigation of Normal Force Distributions and Wake Vortex Characteristics of Bodies of Revolution at Supersonic Speeds," J. Aero. Sci., Vol. 26 (1959), pp. 155-168.

[3]Tinling, B. E. and C. Q. Allen, "An Investigation of the Normal Force and Vortex Wake Characteristics of an Ogive Cylinder," NASA TN D-1927, April 1962.

[4]Fiechter, M., "Deutch-Franzosischeo Forschungs Institut," Saint-Louis (I.S.L.) Bericht, 10/66, 1966.

[5]Grosche, F. R., "Wind Tunnel Investigation of the Vorten System Near an Inclined Body of Revolution With and Without Wings," AGARD-CP-71-71, Aerodynamic Interference, January 1971.

[6]Fidler, J. E., J. N. Nielsen, and R. G. Schwind, "An Investigation of Slender-Body Wake Vortices," AIAA 15th Aerospace Sciences Meeting, AIAA Paper No. 77-7, Los Angeles, CA January 1977.

[7]Yanta, W. J. and A. B. Wardlaw, "Laser Doppler Velocimeter Measurements of Leeward Flowfields on Slender Bodies at Large Angle of Attack," AIAA Paper No. 77-660, Albuquerque, NM, June 1977.

[8]Owen, F. K. and D. A. Johnson, "Measurements of Unsteady Vortex Flowfields," AIAA Journal, Vol. 18 (1980), pp. 1173-1179.

[9]Thompson, K. D. and D. F. Morrison, "The Spacing, Position, and Strength of Vortices in the Wake of Slender Cylindrical Bodies at Large Incidence," J. Fluid Mech., Vol. 50 (1971), pp. 751-783.

[10]Lamont, P. J. and B. L. Hunt, "Pressure and Force Distributions on a Sharp-Nosed Circular Cylinder at Large Angles of Inclination to a Uniform Subsonic Stream," J. Fluid Mech., Vol. 76 (1976), pp. 519-559.

[11]Oberkampf, W. L. and T. J. Bartel, "Supersonic Flow Measurements in the Body Vortex Wake of an Ogive Nose Cylinder," AFATL-TR-78-127, Eglin AFB, FL, 1978.

[12]Graham, J. E., W. L. Hankey, and J. S. Shang, "Navier-Stokes Solution of a Slender Body of Revolution at Large Incidence," AIAA 19th Aerospace Sciences Meeting, January 12-15, 1981, St. Louis, MO.

[13]Shang, J., P. Buning, W. Hankey, and M. Wirth, "Performance of a Vectorized Three-Dimensional Navier-Stokes Code on the CRAY-1 Computer," AIAA Journal, Vol. 18 (1980), p. 9.

[14]Calarese, W., "An Experimental Study of Vortex Flow in the Wake of a Slender Body of Revolution at Large Incidence," AIAA 19th Aerospace Sciences Meeting, January 12-15, 1981, St. Louis, MO.

[15]Shivananda, T. P. and W. L. Oberkampf, "Prediction of the Compressible Vortex Wake for Bodies at High Incidence," AIAA 19th Aerospace Sciences Meeting, January 12-15, 1981, St. Louis, MO.

[16]Milne-Thompson, L. M., "Theoretical Hydrodynamics," Fifth Edition, (The MacMillan Co.: New York) 1968, pp. 351-389.

[17]Foppl, L., "Wirbelbewegung hinter einem Kreiszylinder," Sitzungsberichte d. k. Bayer., Akad der Wissensch, Zu Munchen (1913), pp. 1-17.

[18]Muller, W., Zeit fur techn Physik, Vol. 8, (1927), p. 62.

[19]Bickley, W. G., "The Influence of Vortices Upon the Resistance Experienced by Solids Moving Through a Liquid," Proc. Roy. Soc. A, 119 (1928), pp. 146-156.

[20]Fidler, J. E., "Approximate Method for Estimating Wake Vortex Strength", AIAA Journal, Vol. 12 (1974) pp. 633.

[21]Hooker, S. G., "On the Action of Viscosity in Increasing the Spacing Ratio of a Vortex Street", Proc. Roy. So. A., Vol. 154 (1936) pp. 67.

[22]Schaefer, J. W. and Eskinazi, S., "An analysis of the Vortex Street Generated in a Viscous Fluid", J. Fluid Mech., Vol. 6 (1959), pp. 241.

[23]Abernathy, F. H. and Kronauer, R. E., "The Formation of Vortex Streets", J. Fluid Mech., Vol. 13 (1962), pp. 1.

[24] Schlaer, Karl, A. Angew, Math, Vol. 8, (1928), pp. 352.

[25] Rosenhead, L., "The Spread of Vorticity in the Wake Behind a Cylinder", Proc. Roy. Soc. A., Vol. 127, (1930), pp. 590.

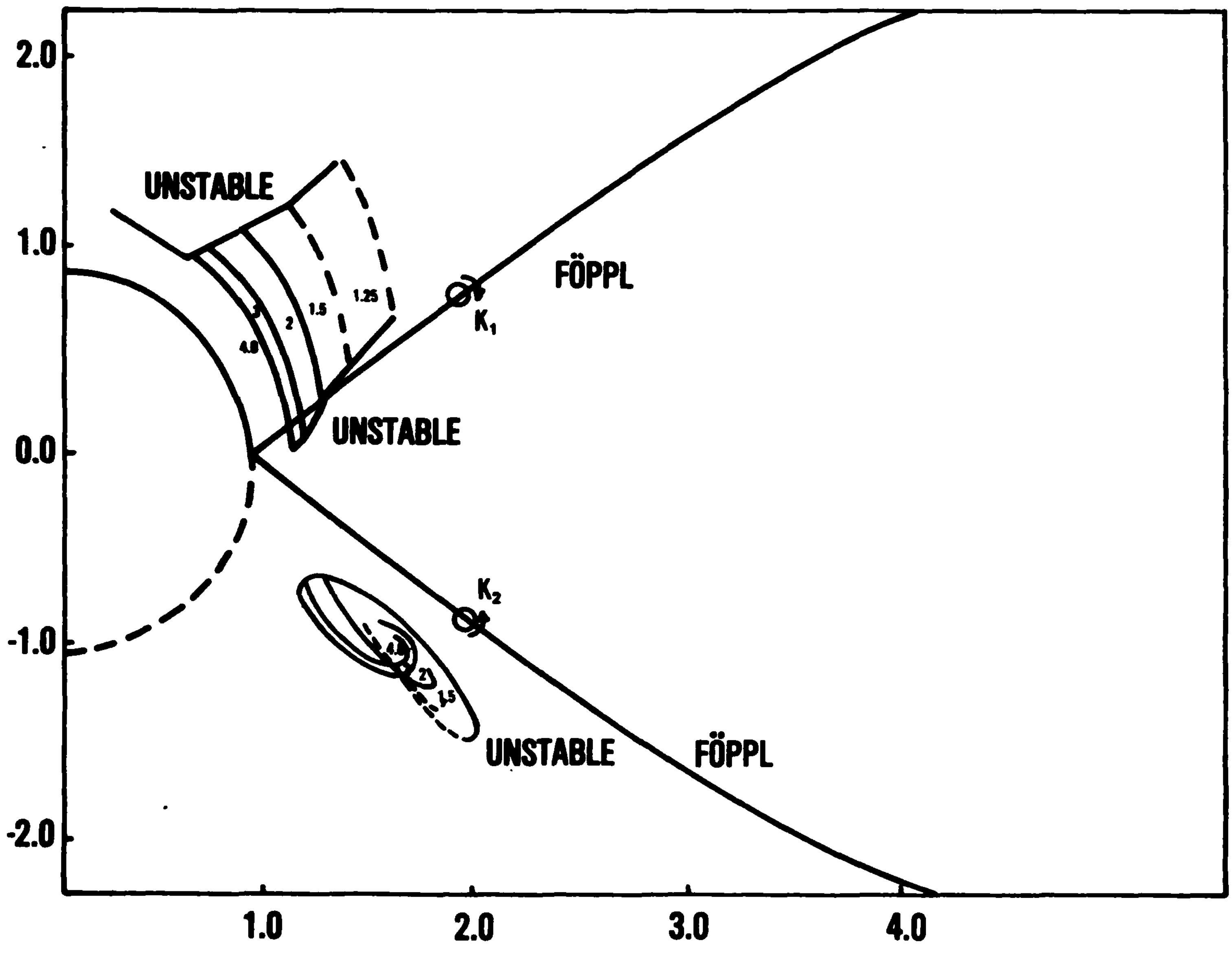

Figure 1 Potential Flow Past a Circular Cylinder and Location of Vortices

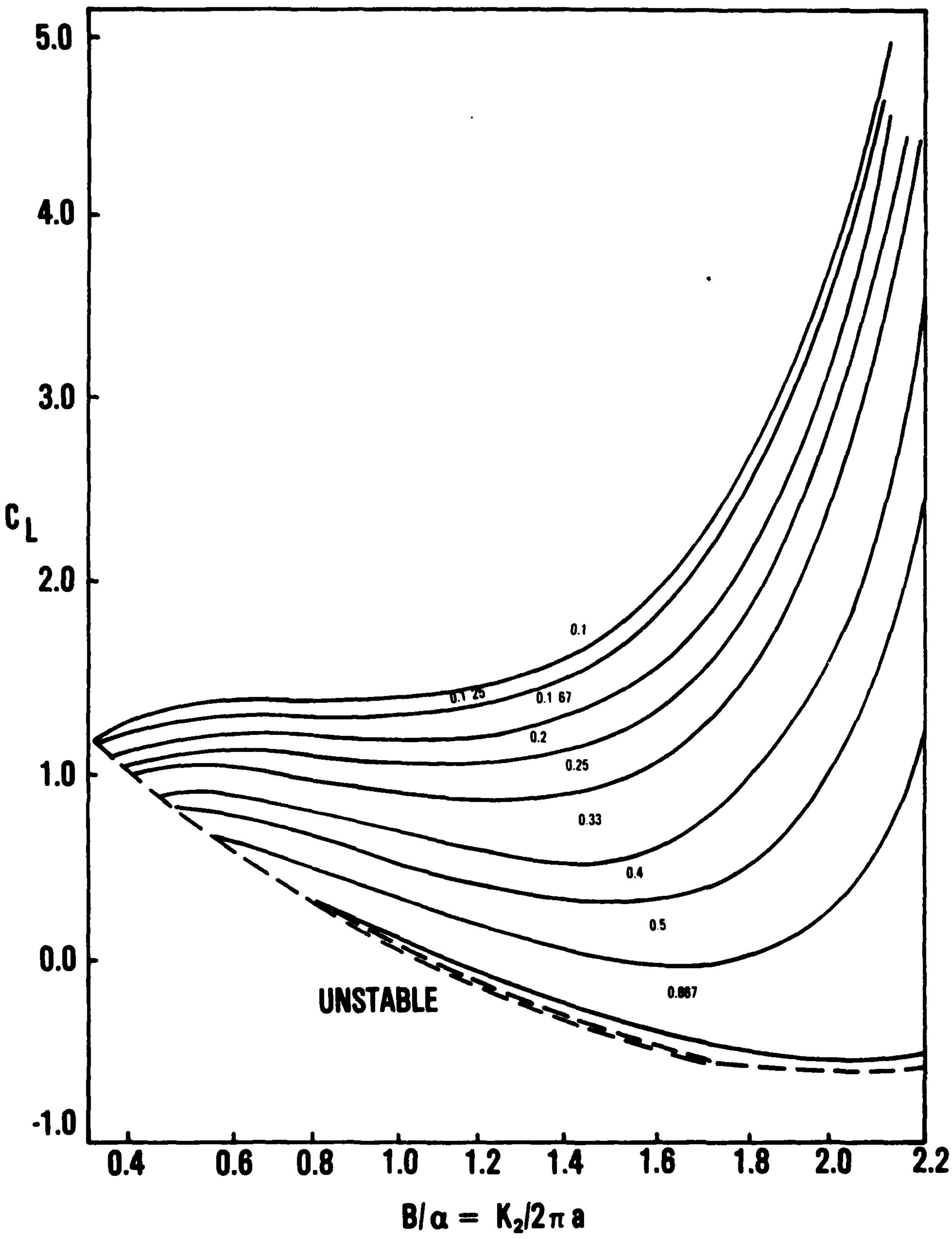

Figure 2 Lift Coefficient C_L vs. (β/α) for various values of $(1/\alpha)$

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